

**What Is Claimed Is:**

1           1.       A method for using a computer system to solve an unconstrained  
2 interval global optimization problem specified by a function  $f$ , wherein  $f$  is a scalar  
3 function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$ , the method comprising:  
4           receiving a representation of the function  $f$  at the computer system;  
5           storing the representation in a memory within the computer system; and  
6           performing an interval global optimization process to compute guaranteed  
7 bounds on a globally minimum value of the function  $f(\mathbf{x})$ ;  
8           wherein performing the interval global optimization process involves,  
9                     applying term consistency over a subbox  $\mathbf{X}$ , and  
10                    excluding any portion of the subbox  $\mathbf{X}$  that violates term  
11                    consistency.

1           2.       The method of claim 1, wherein applying term consistency  
2 involves:  
3           symbolically manipulating an equation within the computer system to  
4 solve for a first term,  $g(\mathbf{x}')$ , thereby producing a modified equation  $g(\mathbf{x}') = h(\mathbf{x})$ ,  
5 wherein the first term  $g(\mathbf{x}')$  can be analytically inverted to produce an inverse  
6 function  $g^{-1}(\mathbf{y})$ ;  
7           substituting the subbox  $\mathbf{X}$  into the modified equation to produce the  
8 equation  $g(\mathbf{X}') = h(\mathbf{X})$ ;  
9           solving for  $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$ ; and  
10           intersecting  $\mathbf{X}'$  with the subbox  $\mathbf{X}$  to produce a new subbox  $\mathbf{X}^+$ ;  
11           wherein the new subbox  $\mathbf{X}^+$  contains all solutions of the equation within  
12 the subbox  $\mathbf{X}$ , and wherein the size of the new subbox  $\mathbf{X}^+$  is less than or equal to  
13 the size of the subbox  $\mathbf{X}$ .

1           3.       The method of claim 1, wherein performing the interval global  
2   optimization process involves:  
3       keeping track of a least upper bound  $f\_bar$  of the function  $f(\mathbf{x})$ ;  
4       removing from consideration any subbox for which  $f(\mathbf{x}) > f\_bar$ ;  
5       applying term consistency to the inequality  $f(\mathbf{x}) \leq f\_bar$  over the subbox  $\mathbf{X}$ ;  
6   and  
7       excluding any portion of the subbox  $\mathbf{X}$  that violates the inequality.

1           4.       The method of claim 1, wherein performing the interval global  
2 optimization process involves:  
3           determining a gradient  $\mathbf{g}(\mathbf{x})$  of the function  $f(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$  includes  
4 components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ );  
5           removing from consideration any subbox for which  $\mathbf{g}(\mathbf{x})$  is bounded away  
6 from zero, thereby indicating that the subbox does not include a global minimum  
7 of  $f(\mathbf{x})$ ; and  
8           applying term consistency to each component  $g_i(\mathbf{x})=0$  ( $i=1, \dots, n$ ) of  $\mathbf{g}(\mathbf{x})=0$   
9 over the subbox  $\mathbf{X}$ ; and  
10          excluding any portion of the subbox  $\mathbf{X}$  that violates a component.

1           5.       The method of claim 1, wherein performing the interval global  
2 optimization process involves:  
3           determining diagonal elements  $H_{ii}(\mathbf{x})$  ( $i=1, \dots, n$ ) of the Hessian of the  
4 function  $f(\mathbf{x})$ ;  
5           removing from consideration any subbox for which a diagonal element of  
6 the Hessian is always negative, which indicates that the  $f$  is not convex and  
7 consequently does not contain a global minimum within the subbox;

8           applying term consistency to each inequality  $H_i(\mathbf{x}) \geq 0$  ( $i=1, \dots, n$ ) over the  
9   subbox  $\mathbf{X}$ ; and  
10         excluding any portion of the subbox  $\mathbf{X}$  that violates an inequality.

1           6.       The method of claim 1, wherein performing the interval global  
2 optimization process involves performing the Newton method, wherein  
3 performing the Newton method involves:  
4           computing the Jacobian  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  of the function  $f$  evaluated as a function of  
5 a point  $\mathbf{x}$  over the subbox  $\mathbf{X}$ ;  
6           computing an approximate inverse  $\mathbf{B}$  of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ ; and  
7           using the approximate inverse  $\mathbf{B}$  to analytically determine the system  
8  $\mathbf{Bg}(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$  is the gradient of the function  $f(\mathbf{x})$ , and wherein  $\mathbf{g}(\mathbf{x})$  includes  
9 components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ ); and  
10          applying term consistency to each component  $(\mathbf{Bg}(\mathbf{x}))_i = 0$  ( $i=1, \dots, n$ ) for  
11 each variable  $x_i$  ( $i=1, \dots, n$ ) over the subbox  $\mathbf{X}$ ; and  
12          excluding any portion of the subbox  $\mathbf{X}$  that violates a component.

1           7. The method of claim 1, further comprising terminating attempts to  
2 further reduce the subbox **X** when:  
3           the width of **X** is less than a first threshold value; and  
4           the magnitude of  $f(\mathbf{X})$  is less than a second threshold value.

1           8.       A computer-readable storage medium storing instructions that  
2   when executed by a computer cause the computer to perform a method for using a  
3   computer system to solve an unconstrained interval global optimization problem  
4   specified by a function  $f$ , wherein  $f$  is a scalar function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots$   
5    $x_n)$ , the method comprising:

6 receiving a representation of the function  $f$  at the computer system;  
7 storing the representation in a memory within the computer system; and  
8 performing an interval global optimization process to compute guaranteed  
9 bounds on a globally minimum value of the function  $f(\mathbf{x})$ ;  
10 wherein performing the interval global optimization process involves,  
11 applying term consistency over a subbox  $\mathbf{X}$ , and  
12 excluding any portion of the subbox  $\mathbf{X}$  that violates term  
13 consistency.

1 9. The computer-readable storage medium of claim 8, wherein  
2 applying term consistency involves:  
3 symbolically manipulating an equation within the computer system to  
4 solve for a first term,  $g(\mathbf{x}')$ , thereby producing a modified equation  $g(\mathbf{x}') = h(\mathbf{x})$ ,  
5 wherein the first term  $g(\mathbf{x}')$  can be analytically inverted to produce an inverse  
6 function  $g^{-1}(\mathbf{y})$ ;  
7 substituting the subbox  $\mathbf{X}$  into the modified equation to produce the  
8 equation  $g(\mathbf{X}') = h(\mathbf{X})$ ;  
9 solving for  $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$ ; and  
10 intersecting  $\mathbf{X}'$  with the subbox  $\mathbf{X}$  to produce a new subbox  $\mathbf{X}^+$ ;  
11 wherein the new subbox  $\mathbf{X}^+$  contains all solutions of the equation within  
12 the subbox  $\mathbf{X}$ , and wherein the size of the new subbox  $\mathbf{X}^+$  is less than or equal to  
13 the size of the subbox  $\mathbf{X}$ .

1 10. The computer-readable storage medium of claim 8, wherein  
2 performing the interval global optimization process involves:  
3 keeping track of a least upper bound  $f\_bar$  of the function  $f(\mathbf{x})$ ;  
4 removing from consideration any subbox for which  $f(\mathbf{x}) > f\_bar$ ;

1           applying term consistency to the inequality  $f(\mathbf{x}) \leq f\_bar$  over the subbox  $\mathbf{X}$ ;  
 2    and  
 3           excluding any portion of the subbox  $\mathbf{X}$  that violates the inequality.

1           11.    The computer-readable storage medium of claim 8, wherein  
 2    performing the interval global optimization process involves:  
 3           determining a gradient  $\mathbf{g}(\mathbf{x})$  of the function  $f(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$  includes  
 4    components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ );  
 5           removing from consideration any subbox for which  $\mathbf{g}(\mathbf{x})$  is bounded away  
 6    from zero, thereby indicating that the subbox does not include a global minimum  
 7    of  $f(\mathbf{x})$ ; and  
 8           applying term consistency to each component  $g_i(\mathbf{x})=0$  ( $i=1, \dots, n$ ) of  $\mathbf{g}(\mathbf{x})=0$   
 9    over the subbox  $\mathbf{X}$ ; and  
 10          excluding any portion of the subbox  $\mathbf{X}$  that violates a component.

1           12.    The computer-readable storage medium of claim 8, wherein  
 2    performing the interval global optimization process involves:  
 3           determining diagonal elements  $H_{ii}(\mathbf{x})$  ( $i=1, \dots, n$ ) of the Hessian of the  
 4    function  $f(\mathbf{x})$ ;  
 5           removing from consideration any subbox for which a diagonal element of  
 6    the Hessian is always negative, which indicates that the  $f$  is not convex and  
 7    consequently does not contain a global minimum within the subbox;  
 8           applying term consistency to each inequality  $H_{ii}(\mathbf{x}) \geq 0$  ( $i=1, \dots, n$ ) over the  
 9    subbox  $\mathbf{X}$ ; and  
 10          excluding any portion of the subbox  $\mathbf{X}$  that violates an inequality.

1           13.     The computer-readable storage medium of claim 8, wherein  
2 performing the interval global optimization process involves performing the  
3 Newton method, wherein performing the Newton method involves:  
4           computing the Jacobian  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  of the function  $f$  evaluated as a function of  
5 a point  $\mathbf{x}$  over the subbox  $\mathbf{X}$ ;  
6           computing an approximate inverse  $\mathbf{B}$  of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ ; and  
7           using the approximate inverse  $\mathbf{B}$  to analytically determine the system  
8  $\mathbf{B}\mathbf{g}(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$  is the gradient of the function  $f(\mathbf{x})$ , and wherein  $\mathbf{g}(\mathbf{x})$  includes  
9 components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ ); and  
10          applying term consistency to each component  $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$  ( $i=1, \dots, n$ ) for  
11 each variable  $x_i$  ( $i=1, \dots, n$ ) over the subbox  $\mathbf{X}$ ; and  
12          excluding any portion of the subbox  $\mathbf{X}$  that violates a component.

1           14.     The computer-readable storage medium of claim 8, wherein the  
2 method further comprises terminating attempts to further reduce the subbox  $\mathbf{X}$   
3 when:  
4           the width of  $\mathbf{X}$  is less than a first threshold value; and  
5           the magnitude of  $f(\mathbf{X})$  is less than a second threshold value.

1           15.     An apparatus that solves an unconstrained interval global  
2 optimization problem specified by a function  $f$ , wherein  $f$  is a scalar function of a  
3 vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ , the apparatus comprising:  
4           a receiving mechanism that is configured to receive a representation of the  
5 function  $f$ ;  
6           a memory for storing the representation; and

7 an interval global optimization mechanism that is configured to perform  
 8 an interval global optimization process to compute guaranteed bounds on a  
 9 globally minimum value of the function  $f(\mathbf{x})$ ;  
 10 a term consistency mechanism within the interval global optimization  
 11 mechanism that is configured to,  
 12 apply term consistency over a subbox  $\mathbf{X}$ , and to  
 13 exclude any portion of the subbox  $\mathbf{X}$  that violates term  
 14 consistency.

1 16. The apparatus of claim 15, wherein the term consistency  
 2 mechanism includes:  
 3 a symbolic manipulation mechanism that is configured to symbolically  
 4 manipulate an equation within the computer system to solve for a first term,  $g(\mathbf{x}')$ ,  
 5 thereby producing a modified equation  $g(\mathbf{x}') = h(\mathbf{x})$ , wherein the first term  $g(\mathbf{x}')$   
 6 can be analytically inverted to produce an inverse function  $g^{-1}(\mathbf{y})$ ;  
 7 a solving mechanism that is configured to,  
 8 substitute the subbox  $\mathbf{X}$  into the modified equation to  
 9 produce the equation  $g(\mathbf{X}') = h(\mathbf{X})$ , and to  
 10 solve for  $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$ ; and  
 11 an intersecting mechanism that is configured to intersect  $\mathbf{X}'$  with the  
 12 subbox  $\mathbf{X}$  to produce a new subbox  $\mathbf{X}^+$ ;  
 13 wherein the new subbox  $\mathbf{X}^+$  contains all solutions of the equation within  
 14 the subbox  $\mathbf{X}$ , and wherein the size of the new subbox  $\mathbf{X}^+$  is less than or equal to  
 15 the size of the subbox  $\mathbf{X}$ .

1 17. The apparatus of claim 15,  
 2 wherein the interval global optimization mechanism is configured to,

1 keep track of a least upper bound  $f\_bar$  of the function  $f(\mathbf{x})$ ,  
 2 and to  
 3 remove from consideration any subbox for which  
 4  $f(\mathbf{x}) > f\_bar$ ;  
 5 wherein the term consistency mechanism is configured to,  
 6 apply term consistency to the inequality  $f(\mathbf{x}) \leq f\_bar$  over  
 7 the subbox  $\mathbf{X}$ , and to  
 8 exclude any portion of the subbox  $\mathbf{X}$  that violates the  
 9 inequality.

1 18. The apparatus of claim 15,  
 2 wherein the interval global optimization mechanism is configured to,  
 3 determine a gradient  $\mathbf{g}(\mathbf{x})$  of the function  $f(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$   
 4 includes components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ ), and to  
 5 remove from consideration any subbox for which  $\mathbf{g}(\mathbf{x})$  is  
 6 bounded away from zero, thereby indicating that the subbox does  
 7 not include a global minimum of  $f(\mathbf{x})$ ; and  
 8 wherein the term consistency mechanism is configured to,  
 9 apply term consistency to each component  $g_i(\mathbf{x})=0$   
 10 ( $i=1, \dots, n$ ) of  $\mathbf{g}(\mathbf{x})=0$  over the subbox  $\mathbf{X}$ , and to  
 11 exclude any portion of the subbox  $\mathbf{X}$  that violates a  
 12 component.

1 19. The apparatus of claim 15,  
 2 wherein the interval global optimization mechanism is configured to,  
 3 determine diagonal elements  $H_{ii}(\mathbf{x})$  ( $i=1, \dots, n$ ) of the  
 4 Hessian of the function  $f(\mathbf{x})$ , and to



remove from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the  $f$  is not convex and consequently does not contain a global minimum within the subbox; and wherein the term consistency mechanism is configured to, apply term consistency to each inequality  $H_{ii}(\mathbf{x}) \geq 0$  ( $i=1, \dots, n$ ) over the subbox  $\mathbf{X}$ , and to exclude any portion of the subbox  $\mathbf{X}$  that violates an inequality.

20. The apparatus of claim 15, further comprising a Newton mechanism within the interval global optimization mechanism that is configured to:

compute the Jacobian  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  of the function  $f$  evaluated as a function of a point  $\mathbf{x}$  over the subbox  $\mathbf{X}$ ;

compute an approximate inverse  $\mathbf{B}$  of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ ; and to

using the approximate inverse  $\mathbf{B}$  to analytically determine the system  $\mathbf{B}\mathbf{g}(\mathbf{x})$ , wherein  $\mathbf{g}(\mathbf{x})$  is the gradient of the function  $f(\mathbf{x})$ , and wherein  $\mathbf{g}(\mathbf{x})$  includes components  $g_i(\mathbf{x})$  ( $i=1, \dots, n$ );

wherein the term consistency mechanism is configured to,

apply term consistency to each component  $(\mathbf{Bg}(\mathbf{x}))_i = 0$

$(i=1, \dots, n)$  for each variable  $x_i$   $(i=1, \dots, n)$  over the subbox  $\mathbf{X}$ , and to

exclude any portion of the subbox  $\mathbf{X}$  that violates term

consistency.

1           21. The apparatus of claim 15, further comprising a termination  
2 mechanism that is configured to terminate attempts to further reduce the subbox **X**  
3 when:  
4           the width of **X** is less than a first threshold value; and  
5           the magnitude of  $f(\mathbf{X})$  is less than a second threshold value.